

# Delay-Guaranteed Cross-Layer Scheduling in Multi-Hop Wireless Networks

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**Abstract**—In this paper, we propose a cross-layer scheduling algorithm that achieves a throughput “ $\epsilon$ -close” to the optimal throughput in multi-hop wireless networks with a tradeoff of  $O(\frac{1}{\epsilon})$  in delay guarantees. The algorithm aims to solve a joint congestion control, routing, and scheduling problem in a multi-hop wireless network while satisfying per-flow average end-to-end delay guarantees and minimum data rate requirements. This problem has been solved for both backlogged as well as arbitrary arrival rate systems. Moreover, we discuss the design of a class of low-complexity suboptimal algorithms, effects of delayed feedback on the optimal algorithm, and extensions of the proposed algorithm to different interference models with arbitrary link capacities.

## I. INTRODUCTION

Cross-layer design of congestion control, routing and scheduling algorithms with Quality of Service (QoS) guarantees is one of the most challenging topics in wireless networking. The back-pressure algorithm first proposed in [1] and its extensions have been widely employed in developing throughput optimal dynamic resource allocation and scheduling algorithms for wireless systems. Back-pressure-based scheduling algorithms have also been employed in wireless networks with time-varying channels [2][3][4]. Congestion controllers at the transport layer have assisted the cross-layer design of scheduling algorithms in [5][6][7], so that the admitted arrival rate is guaranteed to lie within the network capacity region. Low-complexity distributed algorithms have been proposed in [8][9][10][11]. Algorithms adapted to clustered networks have been proposed in [12] to reduce the number of queues maintained in the network. However, delay-related investigations are not included in these works.

In this paper, we propose a cross-layer algorithm to achieve *guaranteed throughput* while satisfying network QoS requirements. Specifically, we construct two virtual queues, i.e., a *virtual queue at transport layer* and a *virtual delay queue*, to *guarantee average end-to-end delay bounds*. Moreover, we construct a *virtual service queue* to *guarantee the minimum data rate required by individual network flows*. Our cross-layer design includes a congestion controller for the input rate to the virtual queue at transport layer, as well as a joint policy for packet admission, routing, and resource scheduling. We show that our algorithm can achieve a throughput arbitrarily close to the optimal. In addition, the algorithm exhibits a tradeoff of

$O(\frac{1}{\epsilon})$  in the delay bound, where  $\epsilon$  denotes the distance from the optimal throughput.

Our main algorithm is further extended: (1) to a set of low-complexity suboptimal algorithms; (2) from a model with constantly-backlogged sources to a model with sources of arbitrary input rates at transport layer; (3) to an algorithm employing delayed queue information; and (4) from a node-exclusive model with constant link capacities to a model with arbitrary link capacities and interference models over fading channels.

The rest of the paper is organized as follows: Section II discusses the related work. In Section III, the network model is presented, followed by corresponding approaches for the considered multi-hop wireless networks. In Section IV, the optimal cross-layer control and scheduling algorithm is described, and its performance analyzed. In Section V, we provide a class of feasible suboptimal algorithms, consider sources with arbitrary arrival rates at transport layer, employ delayed queue information in the scheduling algorithm, and extend the model to arbitrary link capacities and interference models over fading channels. We present numerical results in Section VI. Finally, we conclude our work in Section VII.

## II. RELATED WORK

Delay issues in single-hop wireless networks have been addressed in [13]–[21]. Especially, the scheduling algorithm in [18] provides a throughput-utility that is inversely proportional to the delay guarantee. Authors of [19] have obtained delay bounds for two classes of scheduling policies. A random access algorithm is proposed in [20] for lattice and torus interference graphs, which is shown to achieve order-optimal delay in a distributed manner with optimal throughput. But these works are not readily extendable to multi-hop wireless networks, where additional arrivals from neighboring nodes and routing must be considered. Delay analysis for multi-hop networks with fixed-routing is provided in [22]. Delay-related scheduling in multi-hop wireless networks have been proposed in [23][24][25][26][27]. However, none of the above-mentioned works provide explicit end-to-end delay guarantees.

There are several works aiming to address end-to-end delay or buffer occupancy guarantees in multi-hop wireless networks. Worst-case delay is guaranteed in [28] with a packet dropping mechanism. However, dropped packets are

not compensated or retransmitted with the algorithm of [28], which may lead to restrictions in its practical implementations. A low-complexity cross-layer fixed-routing algorithm is developed in [29] to guarantee order-optimal average end-to-end delay, but only for half of the capacity region. A scheduling algorithm for finite-buffer multi-hop wireless networks with fixed routing is proposed in [30] and is extended to adaptive-routing with congestion controller in [31]. Specifically, the algorithm in [31] guarantees  $O(\frac{1}{\epsilon})$ -scaling in buffer size with a  $\epsilon$ -loss in throughput-utility, but this is achieved at the expense of the buffer occupancy of the source nodes, where *an infinite buffer size* in the network layer is assumed in each source node. This leads to large average end-to-end delay since the network stability is achieved based on queue backlogs at these source nodes.

Compared to the above works, the algorithm presented in this paper develops and incorporates novel virtual queue structures. Different from traditional back-pressure-based algorithms, where the network stability is achieved at the expense of large packet queue backlogs, in our algorithm, “the burden” of actual packet queue backlogs is shared by our proposed virtual queues, in an attempt to guarantee specific delay performances. Specifically, we design a congestion controller for *a virtual input rate* and assign weights in the scheduling policy as a product of actual packet queue backlog and the weighted backlog of a designed virtual queue, which will be introduced in detail in Section IV. As such, the network stabilization is achieved with the help of virtual queue structures that do not contribute to delay in the network. Since *all packet queues* in the network, including those in source nodes, have finite sizes, all average end-to-end delays are bounded independent of length or multiplicity of paths.

### III. NETWORK MODEL

#### A. Network Elements

We consider a time-slotted multi-hop wireless network consisting of  $N$  nodes and  $K$  flows. Denote by  $(m, n) \in \mathcal{L}$  a link from node  $m$  to node  $n$ , where  $\mathcal{L}$  is the set of directed links in the network. Denoting the set of flows by  $\mathcal{F}$  and the set of nodes by  $\mathcal{N}$ , we formulate the network topology  $G = (\mathcal{N}, \mathcal{L})$ . Note that we consider adaptive routing scenario, i.e., the routes of each flow are not determined *a priori*, which is more general than fixed-routing scenario. In addition, we denote the source node and the destination node of a flow  $c \in \mathcal{F}$  as  $b(c)$  and  $d(c)$ , respectively.

We assume that the source node for flow  $c$  is always backlogged at the transport layer. Let the scheduling parameter  $\mu_{mn}^c(t)$  denote the link rate assignment of flow  $c$  for link  $(m, n)$  at time slot  $t$  according to scheduling decisions and let  $\mu_{s(c)b(c)}^c(t)$  denote the admitted rate of flow  $c$  from the transport layer of flow to the source node, where  $s(c)$  denotes the source at the transport layer of flow  $c$ . It is clear that in any time slot  $t$ ,  $\mu_{s(c)n}^c(t) = 0 \ \forall n \neq b(c)$ . For simplicity of analysis, we assume only one packet can be transmitted over a link in one slot, so  $(\mu_{mn}^c(t))$  takes values in  $\{0, 1\}$

$\forall (m, n) \in \mathcal{L}$ . We also assume that  $\mu_{s(c)b(c)}^c(t)$  is bounded above by a constant  $\mu_M \geq 1$ :

$$0 \leq \mu_{s(c)b(c)}^c(t) \leq \mu_M, \ \forall c \in \mathcal{F}, \forall t, \quad (1)$$

i.e., a source node can receive at most  $\mu_M$  packets from the transport layer in any time slot. To simplify the analysis, we prevent looping back to the source, i.e., we impose the following constraints

$$\sum_{m \in \mathcal{N}} (\mu_{mb(c)}^c(t)) = 0 \ \forall c \in \mathcal{F}, \forall t. \quad (2)$$

We employ the node-exclusive model in our analysis, i.e., each node can communicate with at most one other node in a time slot. Note that our model is extended to arbitrary interference models with arbitrary link capacities and fading channels in Section V.D.

We now specify the QoS requirements associated with each flow. The network imposes an *average end-to-end delay threshold*  $\rho_c$  for each flow  $c$ . The end-to-end delay period of a packet starts when the packet is admitted to the source node from the transport layer and ends when it reaches its destination. Note that the delay threshold is a time-averaged upper-bound, not a deterministic one. In addition, each flow  $c$  requires a minimum data rate of  $a_c$  packets per time slot.

#### B. Network Constraints and Approaches

For convenience of analysis, we define  $\mathcal{L}^c \triangleq \mathcal{L} \cup \{(s(c), b(c))\}$ , where the pair  $(s(c), b(c))$  can be considered as a virtual link from transport layer to the source node. We now model queue dynamics and network constraints in the multi-hop network. Let  $U_n^c(t)$  be the backlog of the total amount of flow  $c$  packets waiting for transmission at node  $n$ . For a flow  $c$ , if  $n = d(c)$  then  $U_n^c(t) = 0 \ \forall t$ ; Otherwise, the queue dynamics is as follows:

$$\begin{aligned} U_n^c(t+1) \leq & [U_n^c(t) - \sum_{i:(n,i) \in \mathcal{L}} \mu_{ni}^c(t)]^+ \\ & + \sum_{j:(j,n) \in \mathcal{L}^c} \mu_{jn}^c(t), \text{ if } n \in \mathcal{N} \setminus d(c), \end{aligned} \quad (3)$$

where the operator  $[x]^+$  is defined as  $[x]^+ = \max\{x, 0\}$ . Note that in (3), we ensure that the actual number of packets transmitted for flow  $c$  from node  $n$  does not exceed its queue backlog, since a feasible scheduling algorithm may not depend on the information on queue backlogs. The terms  $\sum_{i:(n,i) \in \mathcal{L}} \mu_{ni}^c(t)$  and  $\sum_{j:(j,n) \in \mathcal{L}^c} \mu_{jn}^c(t)$  represent, respectively, the scheduled departure rate from node  $n$  and the scheduled arrival rate into node  $n$  by the scheduling algorithm with respect to flow  $c$ . Note that (3) is an inequality since the arrival rates from neighbor nodes may be less than  $\sum_j \mu_{jn}^c(t)$  if some neighbor node does not have sufficient number of packets to transmit. Since we employ the node-exclusive model, we have

$$0 \leq \sum_{c \in \mathcal{F}} [\sum_{i:(n,i) \in \mathcal{L}} \mu_{ni}^c(t) + \sum_{j:(j,n) \in \mathcal{L}} \mu_{jn}^c(t)] \leq 1, \ \forall n \in \mathcal{N}. \quad (4)$$

From (1)(2), we also have

$$\sum_{j:(j,n) \in \mathcal{L}^c} \mu_{jn}^c(t) \leq \mu_M, \text{ if } n = b(c), \quad (5)$$

if it is ensured that no packets will be looped back to the source.

Now we construct three kinds of virtual queues, namely, virtual queue  $U_{s(c)}^c(t)$  at transport layer, virtual service queue  $Z_c(t)$  at sources, and virtual delay queue  $X_c(t)$ , to later assist the development of our algorithm:

(1) For each flow  $c$  at transport layer, we construct a virtual queue  $U_{s(c)}^c(t)$  which will be employed in the algorithm proposed in the next section. We denote the virtual input rate to the queue as  $R_c(t)$  at the end of time slot  $t$  and we upper-bound  $R_c(t)$  by  $\mu_M$ . Let  $r_c$  denote the time-average of  $R_c(t)$ . We update the virtual queue as follows:

$$U_{s(c)}^c(t+1) = [U_{s(c)}^c(t) - \mu_{s(c)b(c)}^c(t)]^+ + R_c(t), \quad (6)$$

where the initial  $U_{s(c)}^c(0) = 0$ . Considering the admitted rate  $\mu_{s(c)b(c)}^c(t)$  as the service rate, if the virtual queue  $U_{s(c)}^c(t)$  is stable, then the time-average admitted rate  $\mu_c$  of flow  $c$  satisfies:

$$\mu_c \triangleq \lim_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mu_{s(c)b(c)}^c(\tau) \geq r_c \triangleq \lim_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} R_c(\tau). \quad (7)$$

(2) To satisfy the minimum data rate constraints, we construct a virtual queue  $Z_c(t)$  associated with flow  $c$  as follows:

$$Z_c(t+1) = [Z_c(t) - R_c(t)]^+ + a_c, \quad (8)$$

where the initial  $Z_c(0) = 0$ . Considering  $a_c$  as the arrival rate and  $R_c(t)$  as the service rate, if queue  $Z_c(t)$  is stable, we have:  $r_c \geq a_c$ . Additionally, if  $U_{s(c)}^c(t)$  is stable, then according to (7), the minimum data rate for flow  $c$  is achieved.

(3) To satisfy the end-to-end delay constraints, we construct a virtual delay queue  $X_c(t)$  for any given flow  $c$  as follows:

$$X_c(t+1) = [X_c(t) - \rho_c R_c(t)]^+ + \sum_{n \in \mathcal{N}} U_n^c(t) \quad (9)$$

where the initial  $X_c(0) = 0$ . Considering the packets kept in the network in time slot  $t$ , i.e.,  $\sum_{n \in \mathcal{N}} U_n^c(t)$ , as the arrival rate and  $\rho_c R_c(t)$  as the service rate, and according to queueing theory, if queue  $X_c(t)$  is stable, we have

$$\lim_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \sum_{n \in \mathcal{N}} U_n^c(\tau) \leq \rho_c \lim_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} R_c(\tau) = \rho_c r_c.$$

Furthermore, if  $U_{s(c)}^c(t)$  is stable, then according to (7), we have:

$$\frac{1}{\mu_c} \lim_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \sum_{n \in \mathcal{N}} U_n^c(\tau) \leq \rho_c. \quad (10)$$

In addition, by Little's Theorem, (10) ensures that the average end-to-end delay of flow  $c$  is less than or equal to the threshold  $\rho_c$  with probability (w.p.) 1.

From the above description, we know that the network is *stable* (i.e., each queue at all nodes is stable) and the

average end-to-end delay constraint and minimum data rate requirement are achieved if queues  $U_n^c(t)$  and the three virtual queues are stable for any node and flow, i.e.,

$$\limsup_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}\{X_c(\tau)\} < \infty, \quad \forall c;$$

$$\limsup_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}\{U_n^c(\tau)\} < \infty, \quad \forall n \in \mathcal{N} \cup \{s(c) : c \in \mathcal{F}\};$$

$$\limsup_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}\{Z_c(\tau)\} < \infty, \quad \forall c.$$

Now we define the capacity region of the considered multi-hop network. An arrival rate vector  $(z_c)$  is called *admissible* if there exists some scheduling algorithm (without congestion control) under which the node queue backlogs (not including virtual queues) are stable. We denote  $\Lambda$  to be the capacity region consisting of all admissible  $(z_c)$ , i.e.,  $\Lambda$  consists of all feasible rates stabilizable by some scheduling algorithm *without* considering QoS requirements (i.e., delay constraints and minimum data rate constraints). To assist the analysis in the following sections, we let  $(r_{\epsilon,c}^*)$  denote the solutions to the following optimization problem:

$$\max_{(r_c):(r_c+\epsilon) \in \Lambda} \sum_{c \in \mathcal{F}} r_c$$

$$\text{s.t. } r_c \geq a_c, \quad \forall c \in \mathcal{F}.$$

where  $\epsilon$  is a positive number which can be chosen arbitrarily small. For simplicity of analysis, we assume that  $(a_c)$  is in the interior of  $\Lambda$  and without loss of generality, we assume that there exists  $\epsilon' > 0$  such that  $r_{\epsilon,c}^* \geq a_c + \epsilon' \quad \forall c \in \mathcal{F}$ . According to [32], we have

$$\lim_{\epsilon \rightarrow 0} \sum_{c \in \mathcal{F}} r_{\epsilon,c}^* = \sum_{c \in \mathcal{F}} r_c^*,$$

where  $(r_c^*)$  is the solution to the following optimization:

$$\max_{(r_c):(r_c) \in \Lambda} \sum_{c \in \mathcal{F}} r_c$$

$$\text{s.t. } r_c \geq a_c, \quad \forall c \in \mathcal{F}.$$

#### IV. CONTROL SCHEDULING ALGORITHM FOR MULTI-HOP WIRELESS NETWORKS

Now we propose a control and scheduling algorithm **ALG** for the introduced multi-hop model so that **ALG** stabilizes the network and satisfies the delay constraint and minimum data rate constraint. Given  $\epsilon$ , the proposed **ALG** can achieve a throughput arbitrarily close to  $\sum_{c \in \mathcal{F}} r_{\epsilon,c}^*$ , under certain conditions related to delay constraints which will be later given in Theorem 1.

The optimal algorithm **ALG** consists of two parts: a congestion controller of  $R_c(t)$ , and a joint packet admission, routing and scheduling policy. We propose and analyze the algorithm in the following subsections.

### A. Algorithm Description and Analysis

Let  $q_M \geq \mu_M$  be a control parameter for queue length. We first propose a congestion controller for the input rate of virtual queues at transport layer:

#### 1) Congestion Controller of $R_c(t)$ :

$$\min_{0 \leq R_c(t) \leq \mu_M} R_c(t) \left( \frac{(q_M - \mu_M) U_{s(c)}^c(t)}{q_M} - X_c(t) \rho_c - Z_c(t) - V \right) \quad (11)$$

where  $V > 0$  is a control parameter. Specifically, when  $\frac{q_M - \mu_M}{q_M} U_{s(c)}^c(t) - X_c(t) \rho_c - Z_c(t) - V > 0$ ,  $R_c(t)$  is set to zero; Otherwise,  $R_c(t) = \mu_M$ .

After performing the congestion control, we perform the following joint policy for packet admission, routing and scheduling (abbreviated as *scheduling policy*):

**2) Scheduling Policy:** In each time slot, with the constraints of the underlying interference model as described in Section III including (1)(2)(4), the network solves the following optimization problem:

$$\max_{(\mu_{mn}^c(t))} \sum_{m,n} \mu_{mn}^{c*}(t) w_{mn}(t) \quad (12)$$

$$\text{s.t. } \mu_{mn}^c(t) = 0 \quad \forall c \neq c_{mn}^*(t), \quad \forall (m, n) \in \mathcal{L},$$

$$\mu_{mn}^c(t) = 0 \quad \text{if } n = s(c), \quad \forall c \in \mathcal{F},$$

where  $c_{mn}^*(t)$  and  $w_{mn}(t)$  are defined as follows:

$$c_{mn}^*(t) = \arg \max_{c \in \mathcal{F}} w_{mn}^c(t),$$

$$w_{mn}(t) = [\max_{c \in \mathcal{F}} w_{mn}^c(t)]^+,$$

with weight assignment as follows

$$w_{mn}^c(t) = \begin{cases} \frac{U_{s(c)}^c(t)}{q_M} [U_m^c(t) - U_n^c(t)], & \text{if } (m, n) \in \mathcal{L}, \\ \frac{U_{s(c)}^c(t)}{q_M} [q_M - \mu_M - U_{b(c)}^c(t)], & \text{if } (m, n) = (s(c), b(c)), \\ 0, & \text{otherwise.} \end{cases} \quad (13)$$

In addition, when  $w_{mn}(t) = 0$ , without loss of optimality, we set  $\mu_{mn}^c(t) = 0 \quad \forall c \in \mathcal{F}$  to maximize (12).

Note that  $\mathcal{L} \cup \{(s(c), b(c)) : c \in \mathcal{F}\}$  forms the  $(m, n)$  pairs in  $(\mu_{mn}^c(t))$  over which the optimization (12) is performed. Thus, the optimization is a typical Maximum Weight Matching (MWM) problem. We first decouple flow scheduling from the MWM. Specifically, for each pair  $(m, n)$ , the flow  $c_{mn}^*(t)$  is fixed as the candidate for transmission. We then assign the weight as  $w_{mn}(t)$ . Note also that although similar product form of the weight assignment (13) have been utilized in [30][31], no virtual queues are involved there. Whereas in **ALG**, we assign weights as a product of weighted virtual queue backlog ( $\frac{U_{s(c)}^c(t)}{q_M}$ ) and the actual back-pressure, in an aim to shift the burden of the actual queue backlog to the virtual backlog.

To analyze the performance of the algorithm, we first introduce the following proposition.

**Proposition 1:** Employing **ALG**, each queue backlog in the network has a deterministic worst-case bound:

$$U_n^c(t) \leq q_M, \quad \forall t, \forall n \in \mathcal{N}, \forall c \in \mathcal{F}. \quad (14)$$

*Proof:* We use mathematical induction on time slot in the proof. When  $t = 0$ ,  $U_n^c(0) = 0 \leq q_M \quad \forall n, c$ . In the induction hypothesis, we suppose in time slot  $t$  we have  $U_n^c(t) \leq q_M \quad \forall n, c$ . In the induction step, for any given  $n \in \mathcal{N}$  and  $c \in \mathcal{F}$ , we consider two cases as follows:

(1) We first consider the case when  $n = b(c)$ , i.e., when  $n$  is the source node of flow  $c$ . Since  $U_n^c(t) \leq q_M$  from the induction hypothesis, we further consider two subcases:

- In the first subcase,  $U_{b(c)}^c(t) \leq q_M - \mu_M$ . Then according to the queue dynamics (3) and the inequality (5),  $U_{b(c)}^c(t+1) \leq U_{b(c)}^c(t) + \mu_M \leq q_M$ ;
- In the second subcase,  $q_M - \mu_M < U_{b(c)}^c(t) \leq q_M$ . According to the weight assignment (13), we have  $w_{s(c)b(c)}^c(t) < 0$  which leads to  $\mu_{s(c)b(c)}^c(t) = 0$ . Hence,  $U_{b(c)}^c(t+1) \leq U_{b(c)}^c(t) \leq q_M$  by (2)(3).

(2) In the second case,  $n \neq b(c)$ , i.e.,  $n$  is not the source node of flow  $c$ . Similar to the first case, we further consider the following two subcases:

- In the first subcase,  $U_n^c(t) < q_M$ . Then, since we employ node-exclusive model,  $U_n^c(t+1) \leq U_n^c(t) + 1 \leq q_M$  by (3)(4).
- In the second subcase,  $U_n^c(t) = q_M$ . According to the weight assignment (13) we have  $w_{mn}^c(t) \leq 0 \quad \forall m : (m, n) \in \mathcal{L}$ . Now, for any given node  $m : (m, n) \in \mathcal{L}$ , we have:
  - (i) If  $c \neq c_{mn}^*(t)$ , then by (12),  $\mu_{mn}^c(t) = 0$ ;
  - (ii) Otherwise,  $c = c_{mn}^*(t)$ , which induces  $w_{mn}(t) = [w_{mn}^c(t)]^+ = 0$  and by the scheduling policy,  $\mu_{mn}^c(t) = 0$ .
 Hence  $\mu_{mn}^c(t) = 0 \quad \forall m : (m, n) \in \mathcal{L}$ , and  $U_n^c(t+1) \leq U_n^c(t) = q_M$  by the queue dynamics (3).

The above analysis holds for any given  $n \in \mathcal{N}$  and  $c \in \mathcal{F}$ . Therefore the induction step holds, i.e.,  $U_n^c(t+1) \leq q_M \quad \forall n, c$ , which completes the proof. ■

Now we present our main results in Theorem 1.

**Theorem 1:** Given that

$$q_M > \frac{2N - 1 + \mu_M^2}{2\epsilon} + \mu_M \quad \text{and} \quad \rho_c > \frac{Nq_M}{r_{\epsilon,c}^*} \quad \forall c \in \mathcal{F}, \quad (15)$$

**ALG** can achieve a throughput

$$\liminf_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \sum_{c \in \mathcal{F}} \mathbb{E}\{R_c(\tau)\} \geq \sum_{c \in \mathcal{F}} r_{\epsilon,c}^* - \frac{B}{V}, \quad (16)$$

where  $B \triangleq \frac{1}{2}NKq_M\mu_M + K\frac{q_M - \mu_M}{q_M}\mu_M^2 + \frac{1}{2}\mu_M^2 \sum_{c \in \mathcal{F}} \rho_c^2 + \frac{1}{2}KN^2q_M^2 + \frac{1}{2}K\mu_M^2 + \frac{1}{2}K \sum_{c \in \mathcal{F}} a_c^2$ .

In addition, **ALG** ensures that the virtual queues have a time-averaged bound:

$$\limsup_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \sum_{c \in \mathcal{F}} \mathbb{E}\{U_{s(c)}^c(\tau) + X_c(\tau) + Z_c(\tau)\} \leq \frac{B'}{\delta}, \quad (17)$$



where  $B' \triangleq B + VB_R$ , with  $B_R$  and  $\delta$  constant positive numbers given in the next subsection.

*Remark 1 (Network Stability):* The inequalities (14) from Proposition 1 and (17) from Theorem 1 indicate that **ALG** stabilizes the actual and virtual queues. As an immediate result, **ALG** stabilizes the network and satisfies the average end-to-end delay constraint and the minimum data rate requirement. In addition, Proposition 1 states that the actual queues are *deterministically* bounded by  $q_M$ , which ensures finite buffer sizes for all queues in the network, including those in source nodes.

*Remark 2 (Optimal Utility and Delay Analysis):* Since  $(U_{s(c)}^c(t))$  are stable, the inequality (16) gives a lower-bound on the throughput that **ALG** can achieve. Given some  $\epsilon > 0$ , since  $B$  is independent of  $V$ , (16) also ensures that **ALG** can achieve a throughput arbitrarily close to  $\sum_{c \in \mathcal{F}} r_{\epsilon, c}^*$ . When  $\epsilon$  tends to 0, **ALG** can achieve a throughput arbitrarily close to the optimal value  $\sum_{c \in \mathcal{F}} r_c^*$  with the tradeoff in queue backlog upper-bound  $q_M$  and the delay constraints  $(\rho_c)$ , both of which are lower-bounded by the reciprocal terms of  $\epsilon$  as shown in (15) in Theorem 1. *In other words, the average end-to-end delay bound is of order  $O(\frac{1}{\epsilon})$ .* We note that in **ALG**, the control parameter  $V$ , which is typically chosen to be large, does not affect the actual queue backlog upper-bound or the average end-to-end delay bound, but only affects the upper-bound of the virtual queue backlogs (shown in (17)). In comparison, in the algorithm proposed in [31], the authors show that the internal buffer size is deterministically bounded with order  $O(\frac{1}{\epsilon})$ , but *at the expense of* the buffer occupancy at source nodes which is of order  $O(V)$ , where  $V$  has to be large enough for their algorithm to approach  $\sum_{c \in \mathcal{F}} r_{\epsilon, c}^*$ . This design assumes an *infinite buffer size* at source nodes and typically results in congestion at the source nodes as shown in the simulation results in [31], which further induces an unguaranteed and large average end-to-end delay. Moreover, one can expect that there are no buffer-size guarantees for single-hop flows by employing the algorithm in [31]. In contrast, in our proposed **ALG**, we shift “the burden of  $V$ ” from actual queues to virtual queues and ensure that the average end-to-end delay constraints are satisfied with finite buffer sizes for all actual *packet* queues.

*Remark 3 (Implementation Issues):* To update the virtual queue  $X_c(t)$  and perform the  $R_c(t)$  congestion controller at the transport layer, the queue backlog information of flow  $c$  is crucial. This information can be collected back to the source node by piggy-backing it on ACK from each node. In order to account for such delay of queue backlog information, the  $R_c(t)$  congestion controller (11) of the algorithm can employ delayed queue backlog of  $X_c(t)$ . Similarly, delayed queue backlog information of  $U_{s(c)}^c(t)$  can be employed at the weight assignment (13) of the scheduling policy. The modified algorithm and its validity are further discussed in Section V.C. By employing delayed queue backlog information, we can extend the algorithm to distributed implementation in much the same way as in [8][11] to achieve a *fraction* of the optimal throughput. In order to achieve a through-

put arbitrarily close to the optimal value with distributed implementation, we can employ random access techniques [37][38] in the scheduling policy with fugacities [39] chosen as  $\exp\{\frac{\alpha U_{s(c)}^c(t)[U_m^c(t) - U_n^c(t)]^+}{q_M}\}$  for each link  $(m, n) \in \mathcal{L}$ , where  $\bar{U}_{s(c)}^c(t)$  is a local estimate (e.g., delayed information) of  $U_{s(c)}^c(t)$  and  $\alpha$  a positive weight. It can be shown that the distributed algorithm can still achieve an average end-to-end delay of order  $O(\frac{1}{\epsilon})$  with the time-scale separation assumption [20][36][37].<sup>1</sup> A variation of such distributed implementation in single-hop networks can be found in our recent work [40].

We prove Theorem 1 in the following subsection.

### B. Proof of Theorem 1

Before we proceed, we present the following lemmas which will assist us in proving Theorem 1.

*Lemma 1:* For nonnegative numbers  $A_1, A_2, A_3, Q \in \mathbb{R}$  such that  $Q \leq [A_1 - A_2]^+ + A_3$ , we have  $Q^2 \leq A_1^2 + A_2^2 + A_3^2 - 2A_1(A_2 - A_3)$ .

The proof of Lemma 1 is trivial and omitted. We will later use Lemma 1 to simplify virtual queue dynamics.

*Lemma 2:* For any feasible rate vector  $(\theta_c) \in \Lambda$  with  $\theta_c \geq a_c \forall c \in \mathcal{F}$ , there exists a stationary randomized algorithm STAT that stabilizes the network with input rate vector  $(\mu_{s(c)b(c)}^{STAT}(t))$  and scheduling parameters  $(\mu_{mn}^{c, STAT}(t))$  independent of queue backlogs, such that the expected admitted rates are:

$$\mathbb{E}\{\mu_{s(c)b(c)}^{c, STAT}(t)\} = \theta_c, \forall t, \forall c \in \mathcal{F}.$$

In addition,  $\forall t, \forall n \in \mathcal{N}, \forall c$ , the flow constraint is satisfied:

$$\mathbb{E}\left\{\sum_{i:(n,i) \in \mathcal{L}} \mu_{ni}^{c, STAT}(t) - \sum_{j:(j,n) \in \mathcal{L}^c} \mu_{jn}^{c, STAT}(t)\right\} = 0.$$

Note that it is not necessary for the randomized algorithm STAT to satisfy the average end-to-end delay constraints. Similar formulations of STAT and their proofs have been given in [5] and [6], so we omit the proof of Lemma 2 for brevity.

*Remark 4:* According to the STAT algorithm in Lemma 2, we assign the input rates of the virtual queues at transport layer as  $R_c^{STAT}(t) = \mu_{s(c)b(c)}^{c, STAT}(t)$ . Thus, we also have  $\mathbb{E}\{R_c^{STAT}(t)\} = \theta_c$ . According to the update equation (6), it is easy to show that the virtual queues under STAT are bounded above by  $\mu_M$  and the time-average of  $R_c^{STAT}(t)$  satisfies:  $r_c^{STAT} = \theta_c$ . Note that  $(\theta_c)$  can take values as  $(r_{\epsilon, c}^*)$  or  $(r_{\epsilon, c}^* + \epsilon)$  or  $(r_{\epsilon, c}^* - \frac{1}{2}\epsilon')$ , where we recall  $(r_{\epsilon, c}^* + \epsilon) \in \Lambda$  and  $r_{\epsilon, c}^* \geq a_c + \epsilon' \forall c \in \mathcal{F}$ .

To prove Theorem 1, we first let  $\mathbf{Q}(t) = ((U_n^c(t)), (U_{s(c)}^c(t)), (X_c(t)), (Z_c(t)))$  and define the Lyapunov function  $L(\mathbf{Q}(t))$  as follows:

$$\begin{aligned} L(\mathbf{Q}(t)) = & \frac{1}{2} \left\{ \sum_{c \in \mathcal{F}} \frac{q_M - \mu_M}{q_M} U_{s(c)}^c(t)^2 + \sum_{c \in \mathcal{F}} X_c(t)^2 \right. \\ & \left. + \sum_{c \in \mathcal{F}} Z_c(t)^2 + \sum_{c \in \mathcal{F}} \sum_{n \in \mathcal{N}} \frac{1}{q_M} U_n^c(t)^2 U_{s(c)}^c(t) \right\}. \end{aligned} \quad (18)$$

<sup>1</sup>Note that the random access works cited above either do not provide delay guarantees or are not readily extended to multi-hop settings.

It is obvious that  $L(\mathbf{Q}(0)) = 0$ . We denote the Lyapunov drift by

$$\Delta(t) = \mathbb{E}\{L(\mathbf{Q}(t+1)) - L(\mathbf{Q}(t)) | \mathbf{Q}(t)\}. \quad (19)$$

From the queue dynamics (3)(6), we have:

$$\begin{aligned} & \sum_{c \in \mathcal{F}} \sum_{n \in \mathcal{N}} \frac{1}{q_M} U_n^c(t+1)^2 U_{s(c)}^c(t+1) \\ & \leq \sum_{c \in \mathcal{F}} \frac{1}{q_M} (R_c(t) + U_{s(c)}^c(t)) \sum_{n \in \mathcal{N}} U_n^c(t+1)^2 \\ & \leq \mu_M q_M N K + \sum_{c \in \mathcal{F}} \frac{1}{q_M} U_{s(c)}^c(t) \sum_{n \in \mathcal{N}} \{U_n^c(t)^2 \\ & \quad + (\sum_{i: (n,i) \in \mathcal{L}} \mu_{ni}^c(t))^2 + (\sum_{j: (j,n) \in \mathcal{L}^c} \mu_{jn}^c(t))^2 \\ & \quad - 2U_n^c(t)(\sum_i \mu_{ni}^c(t) - \sum_j \mu_{jn}^c(t))\}, \end{aligned} \quad (20)$$

where we recall that  $R_c(t) \leq \mu_M$  and we employ Lemma 1 to deduce the second inequality.

From (20), we have

$$\begin{aligned} & \frac{1}{2} (\sum_{c \in \mathcal{F}} \sum_{n \in \mathcal{N}} \frac{1}{q_M} (U_n^c(t+1)^2 U_{s(c)}^c(t+1) \\ & \quad - U_n^c(t)^2 U_{s(c)}^c(t))) \\ & \leq \frac{1}{2} \sum_{c \in \mathcal{F}} \frac{(2N-1 + \mu_M^2) U_{s(c)}^c(t)}{q_M} + \frac{1}{2} N K q_M \mu_M \\ & \quad - \sum_{c \in \mathcal{F}} \sum_{n \in \mathcal{N}} \frac{U_n^c(t) U_{s(c)}^c(t)}{q_M} \\ & \quad \left( \sum_{j: (n,j) \in \mathcal{L}} \mu_{nj}^c(t) - \sum_{i: (i,n) \in \mathcal{L}^c} \mu_{in}^c(t) \right), \end{aligned} \quad (21)$$

where we employ the fact deduced from (4)(5) that  $\sum_i \mu_{ni}^c(t) \leq 1$  and  $\sum_j \mu_{jn}^c(t) \leq 1$  when  $n \neq b(c)$  and  $\sum_j \mu_{jn}^c(t) \leq \mu_M$  when  $n = b(c)$ . Note that we use the summation index  $i$  and  $j$  interchangeably for convenience of analysis.

From the queue length dynamics (6) and by employing Lemma 1, we have:

$$\begin{aligned} & \frac{1}{2} \sum_{c \in \mathcal{F}} \frac{q_M - \mu_M}{q_M} (U_{s(c)}^c(t+1)^2 - U_{s(c)}^c(t)^2) \\ & \leq \frac{1}{2} \sum_{c \in \mathcal{F}} \frac{q_M - \mu_M}{q_M} (\mu_{s(c)b(c)}^c(t)^2 + R_c(t)^2 \\ & \quad - 2U_{s(c)}^c(t)(\mu_{s(c)b(c)}^c(t) - R_c(t))) \\ & \leq K \frac{q_M - \mu_M}{q_M} \mu_M^2 \\ & \quad - \frac{q_M - \mu_M}{q_M} \sum_{c \in \mathcal{F}} U_{s(c)}^c(t)(\mu_{s(c)b(c)}^c(t) - R_c(t)). \end{aligned} \quad (22)$$

From the virtual queue dynamics (9), we have:

$$\begin{aligned} & \frac{1}{2} \sum_{c \in \mathcal{F}} (X_c(t+1)^2 - X_c(t)^2) \\ & \leq \frac{1}{2} \sum_{c \in \mathcal{F}} (\rho_c^2 R_c(t)^2 + (\sum_{n \in \mathcal{N}} U_n^c(t))^2 \\ & \quad - 2X_c(t)(\rho_c R_c(t) - \sum_{n \in \mathcal{N}} U_n^c(t))) \\ & \leq \frac{1}{2} \mu_M^2 \sum_{c \in \mathcal{F}} \rho_c^2 + \frac{1}{2} K N^2 q_M^2 \\ & \quad - \sum_{c \in \mathcal{F}} X_c(t) \rho_c R_c(t) + N q_M \sum_{c \in \mathcal{F}} X_c(t). \end{aligned} \quad (23)$$

From the virtual queue dynamics (8), we have:

$$\begin{aligned} & \frac{1}{2} \sum_{c \in \mathcal{F}} (Z_c(t+1)^2 - Z_c(t)^2) \\ & \leq \frac{1}{2} \sum_{c \in \mathcal{F}} (R_c(t)^2 + a_c^2 - 2Z_c(t)(R_c(t) - a_c)) \\ & \leq \frac{1}{2} K \mu_M^2 + \frac{1}{2} \sum_{c \in \mathcal{F}} a_c^2 - \sum_{c \in \mathcal{F}} Z_c(t) R_c(t) + \sum_{c \in \mathcal{F}} a_c Z_c(t). \end{aligned} \quad (24)$$

Substituting (21)(22)(23)(24) into the Lyapunov drift (19) and subtracting  $V \sum_c \mathbb{E}\{R_c(t) | \mathbf{Q}(t)\}$  from both sides, we then have:

$$\begin{aligned} & \Delta(t) - V \sum_{c \in \mathcal{F}} \mathbb{E}\{R_c(t) | \mathbf{Q}(t)\} \\ & \leq B + \sum_{c \in \mathcal{F}} \mathbb{E}\{R_c(t) \left( \frac{(q_M - \mu_M) U_{s(c)}^c(t)}{q_M} \right. \\ & \quad \left. - X_c(t) \rho_c - Z_c(t) - V \right) | \mathbf{Q}(t)\} \\ & \quad + N q_M \sum_{c \in \mathcal{F}} X_c(t) + \sum_{c \in \mathcal{F}} a_c Z_c(t) \\ & \quad + \frac{1}{2} \sum_{c \in \mathcal{F}} \frac{(2N-1 + \mu_M^2) U_{s(c)}^c(t)}{q_M} \\ & \quad - \mathbb{E}\left\{ \frac{q_M - \mu_M}{q_M} \sum_{c \in \mathcal{F}} U_{s(c)}^c(t) \mu_{s(c)b(c)}^c(t) \right. \\ & \quad \left. + \sum_{c \in \mathcal{F}} \sum_{n \in \mathcal{N}} \frac{U_n^c(t) U_{s(c)}^c(t)}{q_M} \right. \\ & \quad \left. \left( \sum_{j: (n,j) \in \mathcal{L}} \mu_{nj}^c(t) - \sum_{i: (i,n) \in \mathcal{L}^c} \mu_{in}^c(t) \right) | \mathbf{Q}(t) \right\}. \end{aligned} \quad (25)$$

We can rewrite the last term of RHS of (25) by simple algebra as

$$\begin{aligned} & -\mathbb{E}\left\{ \sum_{c \in \mathcal{F}} \sum_{(m,n) \in \mathcal{L}} \mu_{mn}^c(t) \frac{U_{s(c)}^c(t)}{q_M} (U_m^c(t) - U_n^c(t)) \right. \\ & \quad \left. + \sum_{c \in \mathcal{F}} \mu_{s(c)b(c)}^c(t) \frac{U_{s(c)}^c(t)}{q_M} (q_M - \mu_M - U_{b(c)}^c(t)) | \mathbf{Q}(t) \right\}. \end{aligned} \quad (26)$$

Then, the second term and the last term of the RHS of (25) are minimized by the congestion controller (11) and the

scheduling policy (12), respectively, over a set of feasible algorithms including the stationary randomized algorithm STAT introduced in Lemma 2 and Remark 4. We can substitute into the second term of RHS of (25) a stationary randomized algorithm with admitted arrival rate vector  $(r_{\epsilon,c}^*)$  and into the last term with a stationary randomized algorithm with admitted arrival rate vector  $(r_{\epsilon,c}^* + \epsilon)$ . Thus, we have:

$$\begin{aligned} & \Delta(t) - V \sum_{c \in \mathcal{F}} \mathbb{E}\{R_c(t) | \mathbf{Q}(t)\} \\ & \leq B - V \sum_{c \in \mathcal{F}} r_{\epsilon,c}^* \\ & \quad - \sum_{c \in \mathcal{F}} \frac{U_{s(c)}^c(t)}{q_M} (\epsilon(q_M - \mu_M) - \frac{2N-1+\mu_M^2}{2}) \\ & \quad - \sum_{c \in \mathcal{F}} (r_{\epsilon,c}^* - a_c) Z_c(t) - \sum_{c \in \mathcal{F}} (\rho_c r_{\epsilon,c}^* - N q_M) X_c(t). \end{aligned} \quad (27)$$

When (15) holds, we can find  $\epsilon_1 > 0$  such that  $\epsilon_1 \leq \rho_c r_{\epsilon,c}^* - N q_M \forall c \in \mathcal{F}$  and  $\epsilon_1 \leq \frac{\epsilon(q_M - \mu_M) - \frac{2N-1+\mu_M^2}{2}}{q_M}$ . Recall that  $\epsilon'$  is defined such that  $r_{\epsilon,c}^* \geq a_c + \epsilon' \forall c \in \mathcal{F}$ . Thus, we have:

$$\begin{aligned} & \Delta(t) - V \sum_{c \in \mathcal{F}} \mathbb{E}\{R_c(t) | \mathbf{Q}(t)\} \\ & \leq B - \delta \sum_{c \in \mathcal{F}} (X_c(t) + U_{s(c)}^c(t) + Z_c(t)) - V \sum_{c \in \mathcal{F}} r_{\epsilon,c}^*, \end{aligned} \quad (28)$$

where  $\delta \triangleq \min\{\epsilon_1, \epsilon'\}$ .

We take the expectation with respect to the distribution of  $\mathbf{Q}$  on both sides of (28) and take the time average on  $\tau = 0, \dots, t-1$ , which leads to

$$\begin{aligned} & \frac{1}{t} \mathbb{E}\{L(\mathbf{Q}(t))\} - \frac{V}{t} \sum_{\tau=0}^{t-1} \sum_{c \in \mathcal{F}} \mathbb{E}\{R_c(\tau)\} \\ & \leq B - V \sum_{c \in \mathcal{F}} r_{\epsilon,c}^* \\ & \quad - \frac{\delta}{t} \sum_{\tau=0}^{t-1} \sum_{c \in \mathcal{F}} \mathbb{E}\{X_c(\tau) + U_{s(c)}^c(\tau) + Z_c(\tau)\}. \end{aligned} \quad (29)$$

Since  $\limsup_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \sum_{c \in \mathcal{F}} \mathbb{E}\{R_c(\tau)\}$  is bounded above (say, by a constant  $B_R$  with  $B_R \leq K \mu_M$ ) and  $\mathbb{E}\{L(\mathbf{Q}(t))\}$  is nonnegative, by taking limsup of  $t$  on both sides of (29), we have:

$$\begin{aligned} & \limsup_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \sum_{c \in \mathcal{F}} \mathbb{E}\{X_c(\tau) + U_{s(c)}^c(\tau) + Z_c(\tau)\} \\ & \leq \frac{B}{\delta} + \frac{V}{\delta} [\limsup_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \sum_{c \in \mathcal{F}} \mathbb{E}\{R_c(\tau)\} - \sum_{c \in \mathcal{F}} r_{\epsilon,c}^*] \\ & \leq \frac{B'}{\delta}, \end{aligned} \quad (30)$$

which proves (17).

By taking liminf of  $t$  on both sides of (29), we have

$$\begin{aligned} & \liminf_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \sum_{c \in \mathcal{F}} \mathbb{E}\{R_c(\tau)\} \\ & \geq \frac{\delta}{V} \liminf_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \sum_{c \in \mathcal{F}} \mathbb{E}\{X_c(\tau) + U_{s(c)}^c(\tau) + Z_c(\tau)\} \\ & \quad - \frac{B}{V} + \sum_{c \in \mathcal{F}} r_{\epsilon,c}^*, \end{aligned} \quad (31)$$

which proves (16) since the first term of the RHS of (31) is nonnegative.

## V. FURTHER DISCUSSIONS

### A. Suboptimal Algorithms

Solving MWM optimization problem can be NP-hard depending on the underlying interference model as indicated in [33]. In this section, we introduce a group of suboptimal algorithms that aim to achieve at least a  $\gamma$  fraction of the optimal throughput. We denote the scheduling parameters of suboptimal algorithms by  $(\mu_{mn}^{c, SUB}(t))$ . For convenience, we also denote the scheduling parameters of **ALG** by  $(\mu_{mn}^{c, OPT}(t))$ . Algorithms are called *suboptimal* if the scheduling parameters  $(\mu_{mn}^{c, SUB}(t))$  satisfy the following:

$$\sum_{m,n} \mu_{mn}^{c, SUB}(t) w_{mn}(t) \geq \gamma \sum_{m,n} \mu_{mn}^{c, OPT}(t) w_{mn}(t), \quad (32)$$

where  $\gamma \in (0, 1)$  is constant and we recall that  $c_{mn}^*(t)$  and  $w_{mn}(t)$  are defined in Section IV.A. In addition, the congestion controller of suboptimal algorithms is the same as that of **ALG** (11).

Following the same analysis of **ALG**, Proposition 1 holds for suboptimal algorithms, i.e., the queue backlogs are bounded above by  $q_M$ , and we derive the following theorem:

*Theorem 2:* Given that

$$\begin{aligned} q_M & > \frac{2N-1+\mu_M^2}{2\gamma\epsilon} + \mu_M \text{ and } \rho_c > \frac{Nq_M}{\gamma r_{\epsilon,c}^*} \forall c \in \mathcal{F}, \\ & \exists \epsilon_2 > 0 \text{ s.t. } \gamma r_{\epsilon,c}^* \geq a_c + \epsilon_2 \forall c \in \mathcal{F}, \end{aligned} \quad (33)$$

a suboptimal algorithm ensures that the virtual queues have a time-averaged bound:

$$\limsup_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \sum_{c \in \mathcal{F}} \mathbb{E}\{U_{s(c)}^c(\tau) + X_c(\tau) + Z_c(\tau)\} \leq \frac{\bar{B}}{\delta}, \quad (34)$$

where  $\bar{B} \triangleq B + \gamma V B_R$ . In addition, a suboptimal algorithm can achieve a throughput

$$\liminf_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \sum_{c \in \mathcal{F}} \mathbb{E}\{R_c(\tau)\} \geq \gamma \sum_{c \in \mathcal{F}} r_{\epsilon,c}^* - \frac{B}{V}. \quad (35)$$

*Proof:* The proof is provided in Appendix A. ■

*Remark 5:* From Theorem 2, given an arbitrarily small  $\epsilon$ , we show that a suboptimal algorithm can *at least* achieve a throughput arbitrarily close to a fraction  $\gamma$  of the optimal

results  $\sum_{c \in \mathcal{F}} r_{\epsilon, c}^*$ . Suboptimal algorithms include the well-known Greedy Maximal Matching (GMM) algorithm [34] with  $\gamma = \frac{1}{2}$  as well as the solutions to the maximum weighted independent set (MWIS) optimization problem such as GWMAX and GWMIN proposed in [35] with  $\gamma = \frac{1}{\Delta}$ , where  $\Delta$  is the maximum degree of the network topology  $G$ . The delay bound and throughput tradeoff in Theorem 1 still hold in Theorem 2.

### B. Arbitrary Arrival Rates at Transport Layer

Note that in the previous model description, we assumed that the flow sources are constantly backlogged, that is, the congestion controller (11) can always guarantee  $R_c(t) = \mu_M$  when  $\frac{q_M - \mu_M}{q_M} U_{s(c)}^c(t) - X_c(t)\rho_c - Z_c(t) - V \leq 0$ . In this subsection, we present an optimal algorithm when the flows have arbitrary arrival rates at the transport layer.

Let  $A_c(t)$  denote the arrival rate of flow  $c$  packets at the beginning of the time slot  $t$  at the transport layer. We assume that  $A_c(t)$  is i.i.d. with respect to  $t$  with mean  $\lambda_c$ . For simplicity of analysis, we assume  $(\lambda_c)$  to be in the exterior of the capacity region  $\Lambda$  so that a congestion controller is needed and we assume that  $A_c(t)$  is bounded above by  $\mu_M \forall c \in \mathcal{F}$ .<sup>2</sup> Let  $L_c(t)$  denote the backlog of flow  $c$  data at the transport layer which is updated as follows:

$$L_c(t+1) = \min\{[L_c(t) + A_c(t) - \mu_{s(c)b(c)}^c(t)]^+, L_M\}, \quad (36)$$

where  $L_M \geq 0$  is the buffer size for flow  $c$  at the transport layer. Note that we have  $L_M = 0$  and  $L_c(t) = 0$  if there is no buffer for flow  $c$  at the transport layer.

Following the idea introduced in [5], we construct a virtual queue  $Y_c(t)$  and an auxiliary variable  $v_c(t)$  for each virtual input rate  $R_c(t)$ , with queue dynamics for  $Y_c(t)$  as follows

$$Y_c(t+1) = [Y_c(t) - R_c(t)]^+ + v_c(t), \quad (37)$$

where initially we have  $Y_c(0) = 0$ . The intuition is that  $v_c(t)$  serves as the function of  $R_c(t)$  in congestion controller (11) and we note that when  $Y_c(t)$  is stable, we have  $r_c \geq v_c$ , where  $v_c$  is the time average rate for  $v_c(t)$ , recalling that  $r_c$  is the time average rate for  $R_c(t)$ . Thus, when  $Y_c(t)$  and  $U_{s(c)}^c(t)$  are stable, if we can ensure the value  $\sum_c v_c$  is arbitrarily close to the optimal value  $\sum_c r_{\epsilon, c}^*$ , then so is the throughput  $\sum_c \mu_c$  since  $\mu_c \geq r_c \geq v_c$ .

Now we provide the optimal algorithm for arbitrary arrival rates at the transport layer:

#### 1) Congestion Controller:

$$\min_{0 \leq v_c(t) \leq \mu_M} v_c(t)(\eta Y_c(t) - V), \quad (38)$$

$$\begin{aligned} \min_{R_c(t)} R_c(t) & \left( \frac{q_M - \mu_M}{q_M} U_{s(c)}^c(t) - \eta Y_c(t) - X_c(t)\rho_c - Z_c(t) \right) \\ \text{s.t.} \quad & 0 \leq R_c(t) \leq \min\{L_c(t) + A_c(t), \mu_M\} \end{aligned} \quad (39)$$

where  $\eta > 0$  is a weight associated with the virtual queue  $Y_c(t)$ . Note that (38) and (39) can be solved independently.

<sup>2</sup>Note that our analysis also works for the case when  $A_c(t)$  is bounded above by some constant  $A_M \forall c \in \mathcal{F}$ , where  $A_M \geq \mu_M$ .

Specifically, when  $\eta Y_c(t) - V \geq 0$ ,  $v_c(t)$  is set to zero; Otherwise,  $v_c(t) = \mu_M$ . When  $\frac{q_M - \mu_M}{q_M} U_{s(c)}^c(t) - \eta Y_c(t) - X_c(t)\rho_c - Z_c(t) \geq 0$ ,  $R_c(t)$  is set to zero; Otherwise,  $R_c(t) = \min\{L_c(t) + A_c(t), \mu_M\}$ .

**2) Scheduling Policy:** The scheduling algorithm is the same as that of **ALG** provided in Section IV.B, except for the updated constraints:  $0 \leq \mu_{s(c)b(c)}^c(t) \leq \min\{L_c(t) + A_c(t), \mu_M\}$ .

Since the scheduling policy is not changed, Proposition 1 still holds. And we present a theorem below for the performance of the algorithm:

**Theorem 3:** Given that

$$q_M > \frac{2N - 1 + \mu_M^2}{2\epsilon} + \mu_M \text{ and } \rho_c > \frac{Nq_M}{r_{\epsilon, c}^*} \forall c \in \mathcal{F},$$

the algorithm ensures that the virtual queues have a time-averaged bound:

$$\limsup_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \sum_{c \in \mathcal{F}} \mathbb{E}\{U_{s(c)}^c(\tau) + X_c(\tau) + Z_c(\tau) + Y_c(\tau)\} \leq \frac{B_2}{\delta'},$$

where  $B_2 \triangleq B + K\eta\mu_M^2 + VB_R$  and  $\delta'$  is constant positive number. In addition, the algorithm can achieve a throughput

$$\liminf_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \sum_{c \in \mathcal{F}} \mathbb{E}\{v_c(\tau)\} \geq \sum_{c \in \mathcal{F}} r_{\epsilon, c}^* - \frac{B_1}{V},$$

where  $B_1 \triangleq B + K\eta\mu_M^2$ .

*Proof:* The proof is provided in Appendix B. ■

Theorem 3 shows that optimality is preserved and  $O(\frac{1}{\epsilon})$  delay scaling is kept.

### C. Employing Delayed Queue Backlog Information

Recall that in **ALG**, congestion controller (11) is performed at the transport layer and link weight assignment in (13) is performed locally at each link. Thus, in order to account for the propagation delay of queue information, we employ delayed queue backlog of  $(X_c(t))$  in (11) and employ delayed queue backlog of  $(U_{s(c)}^c(t))$  for links in  $\mathcal{L}$  in (13). Specifically, we rewrite (11) in **ALG** as:

$$\min R_c(t) \left( \frac{(q_M - \mu_M)U_{s(c)}^c(t)}{q_M} - X_c(t-T)\rho_c - Z_c(t) - V \right), \quad (40)$$

where  $T$  is an integer number that is larger than the maximum propagation delay from a source to a node, and we rewrite (13) as:

$$w_{mn}^c(t) = \begin{cases} \frac{U_{s(c)}^c(t-T)}{q_M} [U_m^c(t) - U_n^c(t)], & \text{if } (m, n) \in \mathcal{L}, \\ \frac{U_{s(c)}^c(t)}{q_M} [q_M - \mu_M - U_{b(c)}^c(t)], & \text{if } (m, n) = (s(c), b(c)), \\ 0, & \text{otherwise.} \end{cases} \quad (41)$$

Proposition 1 still holds and we present a theorem for the scheduling algorithm using delayed queue backlog information, which maintains the throughput optimality and  $O(\frac{1}{\epsilon})$  scaling in delay bound:



*Theorem 4:* Given that

$$q_M > \frac{2N - 1 + \mu_M^2}{2\epsilon} + \mu_M \text{ and } \rho_c > \frac{Nq_M}{r_{\epsilon,c}^*} \quad \forall c \in \mathcal{F},$$

the algorithm ensures that the virtual queues have a time-averaged bound:

$$\limsup_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \sum_{c \in \mathcal{F}} \mathbb{E}\{U_{s(c)}^c(\tau) + X_c(\tau) + Z_c(\tau)\} \leq \frac{B_4}{\delta},$$

where  $B_4 \triangleq B_3 + VB_R$  and  $B_3 \triangleq B + KN\mu_M T + Nq_M T\mu_M \rho_c + K\rho_c^2 \mu_M^2 T$ . In addition, the algorithm can achieve a throughput

$$\liminf_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \sum_{c \in \mathcal{F}} \mathbb{E}\{R_c(\tau)\} \geq \sum_{c \in \mathcal{F}} r_{\epsilon,c}^* - \frac{B_3}{V}.$$

*Proof:* The proof is provided in Appendix C. ■

On employing delayed queue backlogs, we can extend the centralized optimization problem (12) to distributed implementations with methods introduced in Remark 3.

#### D. Arbitrary Link Capacities and Arbitrary Interference Models with Fading Channels

Recall that in the model description in Section III, the link capacity is assumed constant (one packet per slot) and node-exclusive model is employed. In this subsection, we extend the model to arbitrary link capacities and arbitrary interference models with fading channels of finite channel states. Thus, instead of (4), we have  $(\mu_{mn}^c(t))_{(m,n) \in \mathcal{L}} \in I(t)$ , where  $I(t)$  is the feasible activation set for time slot  $t$  determined by the underlying interference model and current channel states, with link capacity constraints  $\sum_{c \in \mathcal{F}} \mu_{mn}^c(t) \leq l_{mn}$ , where  $l_{mn}$  is the arbitrarily chosen link capacity for a link  $(m,n) \in \mathcal{L}$ . We define  $l_n \triangleq \max_{(m,n) \in \mathcal{L}} \sum_{c \in \mathcal{F}} \mu_{mn}^c(t)$ . Note that it is clear that  $l_n \leq \sum_{m:(m,n) \in \mathcal{L}} l_{mn}$ . Then we can update the optimization (12) and weight assignment (13), respectively, as follows:

$$\begin{aligned} & \max_{(\mu_{mn}^c(t))} \sum_{m,n} \mu_{mn}^{c^*} (t) w_{mn}(t) \\ \text{s.t. } & (\mu_{mn}^c(t))_{(m,n) \in \mathcal{L}} \in I(t) \text{ and } \mu_{s(c)b(c)}(t) \leq \mu_M \quad \forall c \in \mathcal{L}. \\ & w_{mn}^c(t) = \begin{cases} \frac{U_{s(c)}^c(t)}{q_M} [U_m^c(t) - U_n^c(t) - l_n], & \text{if } (m,n) \in \mathcal{L}, \\ \frac{U_{s(c)}^c(t)}{q_M} [q_M - \mu_M - U_{b(c)}^c(t)], & \\ 0, & \text{if } (m,n) = (s(c), b(c)), \\ & \text{otherwise.} \end{cases} \end{aligned}$$

It is not difficult to check that Proposition 1 still holds with  $q_M \geq \max\{\max_{n \in \mathcal{N}} l_n, \mu_M\}$  and Theorem 1 holds with a different definition of constant  $B$ . The above modified algorithm can be further extended to solve power allocation problems, where we refer interested readers to our recent work [41].

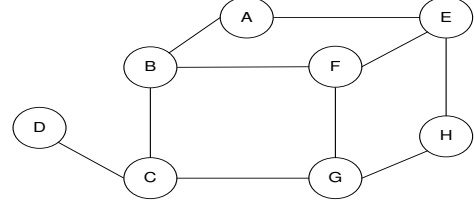


Fig. 1. Network topology for simulations

## VI. NUMERICAL RESULTS

In this section, we present the simulation results for the proposed optimal algorithm **ALG**. Simulations are run in Matlab 2009A with results averaged over  $10^5$  time slots. In the network topology illustrated in Figure 1, there are three source-destination pairs  $(A, G)$ ,  $(D, E)$  and  $(F, H)$  with same Poisson arrival rates and  $\mu_M = 2$ . The required minimum data rate for the three flows are all set to 0.1. We denote by **BP** the back-pressure scheduling algorithm in [1] with a congestion controller in [5], and denote by **Finite Buffer** the cross-layer algorithm developed in [31] with buffer size equal to the queue length limit  $q_M$ . Note that it is shown in simulation results in [31] that Finite Buffer algorithm ensures much smaller internal queue length (of nodes excluding the source node) than BP algorithm (and queue length is related to delay performance). We set the control parameter  $V = 1000$ , where in simulations we find that a higher  $V$  cannot further improve the throughput.

We first illustrate in Table I the throughput optimality of **ALG** when the sources are constantly backlogged. We loosen the delay constraint as  $\rho_c = 30q_M$ . As we increase the control parameter  $q_M$ , the **ALG** achieves a throughput approaching the throughput of BP algorithm which is known to be optimal. We also note that this approximation in throughput results in worse average end-to-end delay performance, which complies with Remark 1.

We then illustrate the throughput and delay tradeoff for both the **ALG** and its corresponding suboptimal GMM algorithm in Figure 2 for the case of arbitrary arrival rates at transport layer with  $L_M = 0$ , where we set  $q_M = 5$  and  $\rho_c = 50$  for each flow  $c$ . Note that this pair of  $q_M$  and  $\rho_c$  shows that the bound in (15) is actually quite loose, and thus our algorithm can achieve better delay performance than stated in (15). Figure 2 shows that the average end-to-end delay under **ALG** is well below the constraint ( $\rho_c = 50$ ) and lower than that under BP and Finite Buffer algorithms. The throughput of **ALG** is close to (although lower than) that of the optimal BP algorithm when arrival rates are small ( $\leq 0.3$ ). Specifically, when the arrival rate is 0.3, **ALG** achieves a throughput 10% more than the GMM algorithm and 9.0% less than BP algorithm, with an average end-to-end delay 35.2% less than the BP algorithm. In the large-input-rate-region ( $> 0.3$ ), we also observe that the delay in both the BP and Finite Buffer algorithm violates the delay constraints. In addition, in the above illustrated scenarios with backlogged and arbitrary arrival rates, the minimum

TABLE I  
THROUGHPUT PERFORMANCE OF **ALG** WHEN SOURCES ARE BACKLOGGED AT THE TRANSPORT LAYER

	<b>ALG</b> ( $\rho_c = 150$ )	<b>ALG</b> ( $\rho_c = 300$ )	<b>ALG</b> ( $\rho_c = 3000$ )	<b>ALG</b> ( $\rho_c = 30000$ )	BP
Throughput (sum for three flows)	0.9368	1.1834	1.2007	1.2305	1.2315
End-to-end delay (averaged over three flows)	45.76	131.47	$1.514 \times 10^3$	$1.3687 \times 10^4$	$3.753 \times 10^4$

arrival rates and average end-to-end delay requirements are satisfied for *individual* flows under **ALG**. As a side note, the average end-to-end delay in all four algorithms in Figure 2 first decreases, which can be explained by the intuition that all the algorithms are based on back-pressure of links (i.e., queue backlog difference of links) and the queue backlog difference tends to be larger for each hop with a larger arrival rate. When arrival rate further increases, congestion level becomes higher since more packets are admitted into the network.

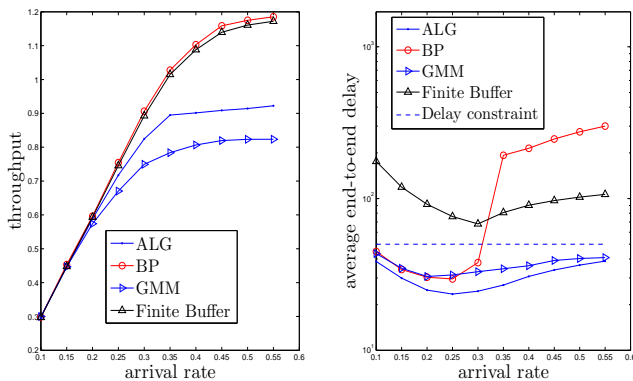


Fig. 2. Throughput and delay tradeoff under Alg. with performances compared to Finite Buffer algorithm and BP algorithm, with varying arrival rates at the transport layer.

## VII. CONCLUSIONS AND FUTURE WORKS

In this paper, we proposed a cross-layer framework which approaches the optimal throughput arbitrarily close for a general multi-hop wireless network. We show a tradeoff between the throughput and average end-to-end delay bound while satisfying the minimum data rate requirements for individual flows.

Our work aims at a better understanding of the fundamental properties and performance limits of QoS-constrained multi-hop wireless networks. While we show an  $O(\frac{1}{\epsilon})$  delay bound with  $\epsilon$ -loss in throughput, how small the actual delay can become still remains elusive, which is dependent on specific network topologies. In our future work, we will investigate the capacity region under end-to-end delay constraints applied to different network topologies. Our future work will also involve power management in the scheduling policies.

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#### APPENDIX A PROOF OF THEOREM 2

*Proof:* Let  $\Delta^{SUB}(t)$  denote the corresponding Lyapunov drift of a suboptimal algorithm which takes the same form as (19). By analyzing (25)(26) which also hold for suboptimal algorithms, we note that the second term of RHS of (25) is always non-positive ensured by the congestion controller (11). Employing (32) to (25)(26), we derive the following

$$\begin{aligned}
& \Delta^{SUB}(t) - V \sum_{c \in \mathcal{F}} \mathbb{E}\{R_c(t)|\mathbf{Q}(t)\} \\
& \leq B + \gamma \sum_{c \in \mathcal{F}} \mathbb{E}\{R_c(t) \left( \frac{(q_M - \mu_M)U_{s(c)}^c(t)}{q_M} \right. \\
& \quad \left. - X_c(t)\rho_c - Z_c(t) - V \right) |\mathbf{Q}(t)\} \\
& \quad + Nq_M \sum_{c \in \mathcal{F}} X_c(t) + \sum_{c \in \mathcal{F}} a_c Z_c(t) \\
& \quad + \frac{1}{2} \sum_{c \in \mathcal{F}} \frac{(2N - 1 + \mu_M^2)U_{s(c)}^c(t)}{q_M} \\
& \quad - \gamma \mathbb{E}\left\{ \frac{q_M - \mu_M}{q_M} \sum_{c \in \mathcal{F}} U_{s(c)}^c(t) \mu_{s(c)b(c)}^{c, SUB}(t) \right. \\
& \quad \left. + \sum_{c \in \mathcal{F}} \sum_{n \in \mathcal{N}} \frac{U_n^c(t) U_{s(c)}^c(t)}{q_M} \right. \\
& \quad \left. \left( \sum_j \mu_{nj}^{c, SUB}(t) - \sum_i \mu_{in}^{c, SUB}(t) \right) |\mathbf{Q}(t)\right\}, \tag{42}
\end{aligned}$$

Following the steps in proving (27), we have from (42)

$$\begin{aligned}
& \Delta(t)^{SUB} - V \sum_{c \in \mathcal{F}} \mathbb{E}\{R_c(t)|\mathbf{Q}(t)\} \\
& \leq B - V \gamma \sum_{c \in \mathcal{F}} r_{\epsilon, c}^* \\
& \quad - \sum_{c \in \mathcal{F}} \frac{U_{s(c)}^c(t)}{q_M} (\gamma \epsilon (q_M - \mu_M) - \frac{2N - 1 + \mu_M^2}{2}) \\
& \quad - \sum_{c \in \mathcal{F}} (\gamma r_{\epsilon, c}^* - a_c) Z_c(t) - \sum_{c \in \mathcal{F}} (\gamma \rho_c r_{\epsilon, c}^* - Nq_M) X_c(t). \tag{43}
\end{aligned}$$

Employing the conditions (33) and following the steps in proving (30) and (31), we can prove Theorem 2. ■

#### APPENDIX B PROOF OF THEOREM 3

Before we proceed to the proof, we extend the stationary randomized algorithm STAT introduced in Lemma 2 and Remark 4. Given  $(\theta_c)$  introduced in Lemma 2 and given flow  $c$  at node  $n$ , recall that  $(A_c(t))$  is i.i.d. with mean  $(\lambda_c)$  and  $(\lambda_c) > (\theta_c)$  element-wise. The flow control for STAT can be given as: Admit  $\mu_{s(c)b(c)}^{c, STAT}(t) = A_c(t)$  w.p.  $\frac{\theta_c}{\lambda_c}$ ; otherwise,  $\mu_{s(c)b(c)}^{c, STAT}(t) = 0$ . Then  $\mathbb{E}\{\mu_{s(c)b(c)}^{c, STAT}(t)\} = \theta_c, \forall t$ . Now take  $v_c^{STAT}(t) = R_c^{STAT}(t) = \mu_{s(c)b(c)}^{c, STAT}(t) \forall c \in \mathcal{F}$ . Then we also have  $\mathbb{E}\{v_c^{STAT}(t)\} = \mathbb{E}\{R_c^{STAT}(t)\} = \theta_c$ . Note that  $R_c^{STAT}(t) \leq A_c(t) \leq \min\{L_c(t) + A_c(t), \mu_M\}$  and  $v_c^{STAT}(t) \leq \mu_M$ .

Now we present the proof.

*Proof:* We define the Lyapunov function as  $L(\mathbf{Q}'(t)) = L(\mathbf{Q}(t)) + \frac{\eta}{2} \sum_{c \in \mathcal{F}} Y_c^2(t)$  and the Lyapunov drift as  $\Delta'(t) = \mathbb{E}\{L(\mathbf{Q}'(t+1)) - L(\mathbf{Q}'(t)) | \mathbf{Q}'(t)\}$ , where  $\mathbf{Q}'(t) = (\mathbf{Q}(t), (Y_c(t)))$ . From the virtual queue dynamics (37) and

Lemma 1, we have

$$\begin{aligned}
& \frac{\eta}{2} \sum_{c \in \mathcal{F}} (Y_c(t+1)^2 - Y_c(t)^2) \\
& \leq \frac{\eta}{2} \sum_{c \in \mathcal{F}} (R_c(t)^2 + v_c(t)^2 - 2Y_c(t)(R_c(t) - v_c(t))) \quad (44) \\
& \leq K\eta\mu_M^2 - \sum_{c \in \mathcal{F}} \eta Y_c(t)(R_c(t) - v_c(t)).
\end{aligned}$$

Following the steps in deriving (25)(26), we have

$$\begin{aligned}
& \Delta'(t) - V \sum_{c \in \mathcal{F}} \mathbb{E}\{v_c(t)|\mathbf{Q}'(t)\} \\
& \leq B_1 + \sum_{c \in \mathcal{F}} \mathbb{E}\{v_c(t)(\eta Y_c(t) - V)|\mathbf{Q}'(t)\} \\
& + \sum_{c \in \mathcal{F}} \mathbb{E}\{R_c(t) \left( \frac{(q_M - \mu_M)U_{s(c)}^c(t)}{q_M} \right. \\
& \quad \left. - \eta Y_c(t) - X_c(t)\rho_c - Z_c(t) \right) |\mathbf{Q}'(t)\} \\
& + Nq_M \sum_{c \in \mathcal{F}} X_c(t) + \sum_{c \in \mathcal{F}} a_c Z_c(t) \\
& + \frac{1}{2} \sum_{c \in \mathcal{F}} \frac{(2N - 1 + \mu_M^2)U_{s(c)}^c(t)}{q_M} \\
& - \mathbb{E}\left\{ \sum_{c \in \mathcal{F}} \sum_{(m,n) \in \mathcal{L}} \mu_{mn}^c(t) \frac{U_{s(c)}^c(t)}{q_M} (U_m^c(t) - U_n^c(t)) \right. \\
& \left. + \sum_{c \in \mathcal{F}} \mu_{s(c)b(c)}^c(t) \frac{U_{s(c)}^c(t)}{q_M} (q_M - \mu_M - U_{b(c)}^c(t)) |\mathbf{Q}'(t)\} \right\},
\end{aligned}$$

The second term, third term and the last term of the RHS of (45) are minimized by the congestion controller (38), (39) and the scheduling policy (12), respectively, over a set of feasible algorithms including the stationary randomized algorithm STAT. Substitute into the second term of RHS of (45) a stationary randomized algorithm with admitted arrival rate vector  $(r_{\epsilon,c}^* - \frac{1}{2}\epsilon')$ , the third term a stationary randomized algorithm with admitted arrival rate vector  $(r_{\epsilon,c}^*)$  and the last term a stationary randomized algorithm with admitted arrival rate vector  $(r_{\epsilon,c}^* + \epsilon)$ . Then, following the steps in proving Theorem 1, we can prove Theorem 3. ■

## APPENDIX C PROOF OF THEOREM 4

*Proof:* According to queue dynamics (6)(9), we obtain

$$\begin{aligned}
U_{s(c)}^c(t) - \mu_M T & \leq U_{s(c)}^c(t - T) \leq U_{s(c)}^c(t) + \mu_M T, \\
X_c(t) - Nq_M T & \leq X_c(t - T) \leq X_c(t) + \rho_c \mu_M T.
\end{aligned} \quad (46)$$

Employing the above inequalities to (25)(26), we have

$$\begin{aligned}
& \Delta(t) - V \sum_{c \in \mathcal{F}} \mathbb{E}\{R_c(t)|\mathbf{Q}(t)\} \\
& \leq B + \sum_{c \in \mathcal{F}} \mathbb{E}\{R_c(t) \left( \frac{(q_M - \mu_M)U_{s(c)}^c(t)}{q_M} \right. \\
& \quad \left. - X_c(t - T)\rho_c - Z_c(t) - V \right) |\mathbf{Q}(t)\} \\
& + Nq_M \sum_{c \in \mathcal{F}} X_c(t) + \sum_{c \in \mathcal{F}} a_c Z_c(t) + K\rho_c^2 \mu_M^2 T + \frac{1}{2} K N \mu_M T \\
& + \frac{1}{2} \sum_{c \in \mathcal{F}} \frac{(2N - 1 + \mu_M^2)U_{s(c)}^c(t)}{q_M} \\
& - \mathbb{E}\left\{ \sum_{c \in \mathcal{F}} \sum_{(m,n) \in \mathcal{L}} \mu_{mn}^c(t) \frac{U_{s(c)}^c(t - T)}{q_M} (U_m^c(t) - U_n^c(t)) \right. \\
& \left. + \sum_{c \in \mathcal{F}} \mu_{s(c)b(c)}^c(t) \frac{U_{s(c)}^c(t)}{q_M} (q_M - \mu_M - U_{b(c)}^c(t)) |\mathbf{Q}(t)\} \right\}.
\end{aligned}$$

The second term and the last term of the RHS of the above inequality are minimized by the congestion controller (40) and the scheduling policy (12) with weight assignment (41), respectively, over a set of feasible algorithms including the stationary randomized algorithm STAT. Substitute into the second term of RHS a stationary randomized algorithm with admitted arrival rate vector  $(r_{\epsilon,c}^*)$  and the last term a stationary randomized algorithm with admitted arrival rate vector  $(r_{\epsilon,c}^* + \epsilon)$ . Then, employing the inequalities (46) and following the steps in proving Theorem 1, we can prove Theorem 4. ■